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A New Approach to Causality Testing

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ABSTRACT – *A new causality test based on Higher Order Cumulants (HOC) is proposed in this paper. The test can be applied on non Gaussian time series. The methodological novelty is the usage of a two- step method based on digital whitening, which is performed by ARMA-HOC filter. To substantiate the method further, an empirical analysis of the relationship between the interest rate spread and real gross domestic product (GDP) growth is presented for the period 1982:q1 -2010:q1. The spread is measured as a difference between 10-year bond yields and three-month Treasury bill rates in the US. The fist step applies ARMA-HOC models to obtain white residuals from a quarterly term spread (TS) and GDP growth. The second step tests the dynamical correlation of TS and GDP growth residuals. The results show that the proposed test can capture the information about non Gaussian properties of the random variables being tested. The test is compared with the Granger-Sims causality test. The paper questions the reliability of the Granger test*

KEY WORDS: *Non Gaussian Time Series, Causality testing, Higher Order Cumulants, Granger-Sims test, Box-Hough test, ARMA-HOC test*

Introduction

The availability of large data sets of high frequency time series in finance and economics has led to the settlement of some old disputes regarding the nature of the data but it also generated new challenges.

A set of properties common across many financial variables, instruments and markets, has been observed and classified in independent studies as "stylized facts". One of the most important stylized properties of asset returns and financial variables in general, besides the absence of correlation, is heavy tails or existence of higher order moments and tail index which is finite and higher than two and less than six (Cont 2001).

The methodology widely used to test the occurrence of causality is known as Granger's methodology. Actually, Wiener was the first to state a causality definition by suggesting that Xt is causal to Y_t if X_t reduces the mean square prediction error of Yt. Granger explored Wiener's definition further. Sims gave content to Granger's definition by assuming that X_t (, Y_t) are jointly covariance stationary Gaussian processes and proving the causality theorem. The theorem states that for X_t and Y_t, having autoregressive representations, Y_t can be expressed as a distributed lag function of current and past X_t with residuals which are not correlated with any values of X_t, past or future, if and only if Y_t does not cause X_t in Granger's terms. The application of the Granger-Sims methodology is usually used with two

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objectives: to test the causality between different economic variables and simultaneously to define lags for which that causality exists. Therefore, while searching for the lag identification, authors are forced to ignore the fact that residuals from the test models might be uncorrelated. Having realized shortcomings of the ad hock filter while applying Granger-Sims tests, Hough and Pierce (1977) introduced the causality test based on the correlation between driving white noises for X_t and Y_t, ut and vt respectfully. Although Box (1970) introduced the idea for the first time, this test has not brought a wide attention in econometrics.

The empirical part of the paper tests causality between GDP growth and the term spread.

In fact, over the last decade empirical researches have demonstrated positive relationship between the slope of the yield curve and real economic growth. The predictive power of the term spread has been recognized beyond the academic research arena. The conference board uses the yield spread in constructing its Index of Leading Indicators. The fact that the yield curve slope changes across the business circle is used by researches which investigated recession and power of the term spread to predict it. The slope of the term structure has often been represented in the economic literature as the spread between long term bonds and short term treasury bills.

The first papers, dealing with the US data, found a significant relationship between the term spread and real activity with lead times between 1 to 8 quarters (Chen (1991), Estrella (1991), Harvey (1995), Dotsey (1998), Bonser (1977), Ang (2003). Guided by the intuition that during recessions, upward sloping yield curves indicate bad times today, but also better times tomorrow, researchers predicted GDP growth using LS regression. Bonser-Neal further established) at what horizons the yield spread best aids in predicting real growth.

On the other side, the cause of a possible relationship between the term structure and GDP growth according to Taylor (1993) is monetary policy reaction function. His model contains Philips curve, the dynamic IS curve, Fisher equation, the expectations hypothesis and a monetary policy rule. Estrella explored the model and found a positive relationship between the spread and GDP growth. Although the results obtained for different periods show strong relationship between the Term spread and GDP growth, they also demonstrate that the relationship might not be stable over time.

The aim of this paper is to propose and to apply the HOC based causality test to investigate a dynamical relationship between the term spread and real GDP growth. The novelty of the paper is the two-step HOC based test, which is based on the assumption that a possible cause of the instability of the relationship are non Gaussian properties of the variables that can be captured by higher order moments-cumulants. In the first step, two time series are whitened using time series models (ARIMA models based on higher order cumulants) in order to obtain the prediction errors known as innovations. In the second step, causality between white innovations is performed using the Pierce & Hough test. This test appeared to be useful in eliminating potential influence of a third, unknown variable and appreciating the fact that Xt might not be the only variable that explains Yt. To sustain the theoretical analysis, the first part of the empirical analysis is done with the US Term Spread (TS) data and real GDP quarterly data. The sample spans the period from 1989: q1 to 2010:q1.

The paper is organized as follows: The second section provides a brief review of the traditional approach to causality testing used in literature so far. The third section introduces the HOC based test. The fourth section contains a statistical data description and empirical results obtained using the HOC test. The last section contains the conclusion.

Problem formulation and methodology

Granger-Sims causality test

The most popular method for testing statistical causality between stock prices and the economy is "Granger-causality" test proposed by C.J.Granger (1969). According to Granger, X causes Y if the past values of X can be used to predict Y more accurately than simply using the past values of Y. In other words, if the past values of X statistically improve the prediction of Y, then we can conclude that "Granger-causes" Y. If the sum of the squared residuals that remain after getting econometric model between Yt and Xt is denoted by SSR, the test gets the form:

$SSR0\ (Y_t / (Y_{t-1} + X_{t-1}) < SSR1\ (Y_t / Y_{t-1})$, if X_t Granger causes Y_t

To compare two variances, the F test is to be used.

It should be pointed out that given the controversy surrounding the Granger causality method, the empirical results and conclusions drawn from them should be considered suggestive rather than absolute. This is especially important in light of the "false signals" that the test has generated in the past.

Box-Hough test

As it was theoretically proven in the literature, the alternative causality test is based on whitening filtration of X_t and Y_t, or by testing "white" residuals of the both variables X_t and Y_t. This test is supposed to eliminate a possibility of having a relationship between two variables when both are driven, or influenced by some third variable. Further ,it was proven by Hough (1977) that if there is a dynamical correlation between Yt prediction errors and past X_t prediction errors we can say that X_t drives or causes Y_t. Vice-versa, if there is a dynamical correlation between Y_t prediction errors and past X_t prediction errors we can say that X_t drives Y_t.If prediction errors of X_t drive Y_t and prediction errors of Y_t drives X_t, there is a feedback between two variables.

Pierce and Haugh have formally defined causality restrictions regarding the correlation coefficient ρ_{uv} between driving white noises for Y_t and X_t, $\mathbf{u}_t$ and $\mathbf{v}_t$:

$\rho_{uv)}(k) >< 0$	For every k>0	X_t causes Y_t
$\rho_{uv}(k) > < 0$	For every k<0	Y_t causes X_t
$\rho_{uv}(0) > < 0$		Instantaneous Causality

Later on, Box and Haugh (1977) proved that ρ_{uv} has an asymptotically normal distribution with variance 1/(n-k), where n is the number of observations and thus enabled causality testing and k being the lag size.

The rational behind this test might be explained by two facts: A dynamical cross correlation between two stationary variables gives false signals about the relationship if the transfer functions of the ARMA models that are used to describe X_t and Y_t are linked; White residuals, from ARMA models, have one more meaning: one step ahead prediction errors for X_t and Y_t, or innovation. Therefore one can say that X_t causes Y_t if X_t innovations cause Y_t innovation.

HOC based test

Let X_t and Y_t be jointly stationary non Gausian processes with finite first, second, third and forth moments that can be treated as outputs from the linear ARIMA filters, whose inputs are white noise signals: $\mathbf{u}_t$ and $\mathbf{v}_t$ respectively:

$$A1(Z)^* DX_t= B1(Z)^* u_t \quad (1)$$

$$A2(Z)^* DY_t= B2(Z)^* v_t \quad (2)$$

Where Z is a backward shift operator : $Y_{t-1}=ZY_t$, $Y_{t-k} =Z^kY_t$, $A(Z) = 1-\alpha_1Z-\alpha_2Z^2 - \dots\alpha_pZ^p$ and $B(Z) = 1-\beta_1Z-\beta_2Z^2 - \dots\beta_qZ^q$ are AR and MA filters of orders p and q respectively, D is the first difference filter, $DY_t = Y_t - Y_{t-1}$, $D^kY_t=Y_t - Y_{t-k}$.

It is worth stressing that the main premises in this methodology is that each stationary time series is treated as the output from AR(p), MA(q) or ARIMA(p,d,q) filter, which has as the input uncorrelated and non Gaussian shocks known as "non Gaussian white noise".

Given the time series Xt and Yt observed at a regular sampling interval it is necessary to define the relationship between them: as Xt causes Yt, Yt causes Xt, feedback or independence. The empirical research problem in this paper is to identify relationship between the TS and the GDP growth.

In this article, ARIMA (p,d,q) time series modeling is based on higher order cumulants .The later type of the model is used since it was found that ignoring non Gaussian nature of both time series significantly reduce the power of the causality test. Nonetheless the cumulants based ARMA estimates are shown to be asymptotically optimal by Friendler B. and Porat B. (1989), the ARMA models based on higher order cumulants have been used so far only in the area of non Gaussian digital signal processing and have not been used in finance and economics due to its numerical complexity.

ARMA parameter estimation using cumulants

Giannakis (1990), was the first to show that the AR parameters of non-Gaussian ARMA digital signals can be calculated using the third- and fourth-order cumulants of the output time series given by:

$$C^3_x(\tau1,\tau2)= (\textstyle\sum(x(t)x(t+\tau_1)x(t+\tau_2))/n, \quad (3)$$

$$C^4(\tau1,\tau2,\tau3,)= (\textstyle\sum(x(t)x(t+\tau_1)x(t+\tau_2)\, x(t+\tau_3))/n -$$

$$-C^2_x(\tau1)\, C_x(\tau2-\tau3) - C^2_x(\tau2)\, C_x(\tau3-\tau1)-C^2_x(\tau3)\, C_x(\tau1-\tau2), \quad (4)$$

where n is a number of observations and where the second-order cumulant $\mathbf{C}^2_x(\circledcirc)$ is just the autococariance function of the time series x_t..

The zero lag cumulant of the order three, $C^3_x(0,0)$ normalized by $\circledcirc_x^3$ is skewness γ^3_x; $C^4_x(0,0,0)$ normalized by $\circledcirc_x^4$ is known as kurtosis γ^4_x.

A new method of the AR parameter estimation for non-Gaussian ARMA (p,q) digital signals is based on the modified Yule-Walker system where autocorrelations are replaced by third or fourth order cumulants (Gianninakis -1990):

$$\sum_{\underline{1=1}}^{p} \alpha i\, C^3(k-i,k-l) \quad = - \quad C^3(k, k-l) \qquad k \geq l \geq q+1 \qquad (5)$$

$$\sum_{\underline{1=1}}^{p} \circledcirc i\, C^4(k-i,k-l, k-m) \quad = - \quad C^4(k, k-l, k-m) \qquad k \geq l \geq m \geq q+1 \qquad (6)$$

Silva Isabel and Silva Edvarda (2006) considered modified Yule-Walker parameter estimation for the pth-order integer-valued autoregressive, INAR(p) process . In particular, the asymptotic distribution of the Yule-Walker estimator was obtained and it was shown that this estimator is asymptotically normally distributed, unbiased and consistent.

The efficient MA parameter estimation can be performed by applying one of the algorithms related to signal processing , for instance, q-slice algorithm (Swami 1989).Q –slice algorithm uses autoregressive residuals calculated after estimating the AR parameters of the ARMA model.

Following up, the impulse response parameters ψi of the pure MA model can be estimated using cumulants (8):

$$x_t = \sum_{0}^{\infty} \psi_j a_{t-j} \qquad i=1.2\ldots\infty \qquad (7)$$

$$\psi j = \frac{\sum^{p} \alpha i\, C^3(q-i,j)}{\sum^{p} \alpha i\, C^3(q-i,0)} \qquad j=1,2\ldots q \qquad (8)$$

Or by using :

$$\psi j = \frac{\sum^{p} \alpha i\, C^4(q-i,j,0)}{\sum^{p} \alpha i\, C^4(q-i,0,0)} \qquad j=1,2\ldots q \qquad (9)$$

The MA parameters of the ARMA model are obtained by means of the well known relationship

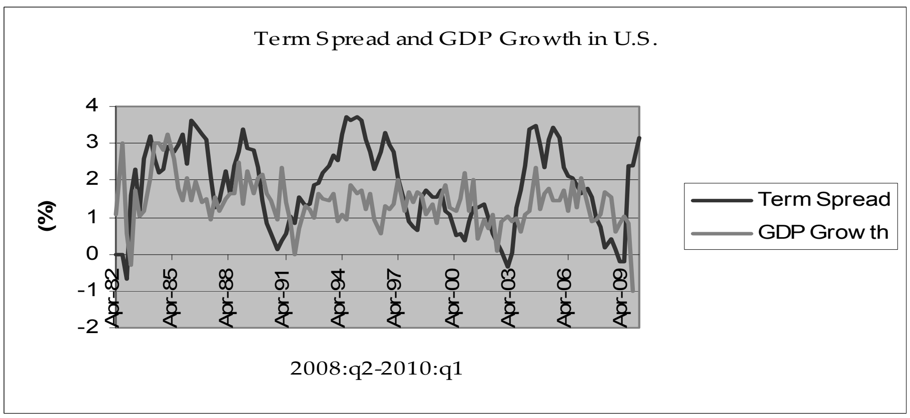
$$\beta_j = \sum_{i=1}^{p} \alpha i \ \psi_{(j-i)} \quad j=1,2\ldots q \tag{10}$$

Data description and Empirical Results

Granger test results

Real GDP data are taken from Bloomberg, 10-year Treasury bonds and three-month treasury bills rates are taken quarterly from the web page economagic.com for the period 1982:q1- 2010 :q1. Figure 1 shows how all variables change.

Figure 1. GDP growth and Interest Rate Yields



Statistical data description is obtained using E-Views program and it is presented in Table 1. Table 1 shows that both variables are non-Gaussian, according to the skewness, kurtosis and the Jarque-Bera test for normality.

Table 1. Data Description

	TSPREAD	GDPCH
Mean	1.910	0.014
Median	1.910	0.014
Maximum	3.730	0.032
Minimum	-0.670	-0.010
Std. Dev.	1.086	0.007
Skewness	-0.181	-0.021
Kurtosis	2.029	5.790
Jarque-Bera	4.970	14.824
Probability	0.083	0.001
Observations	111	111

The results of the Granger causality test between the growth data and the term spread (TS) for the lags l, 2…8 are presented in Table 2. The test shows a feedback relationship between the Term Spread and GDP change for the quarters 1 and 2. It also shows that term spread does Granger cause GDP change across three quarters, while GDP change Granger causes term spread over next two quarters.

Table 2. Granger Causality test results

Sample: 1982Q1 2010Q1				
Lags	Null Hypothesis:	Obs	F-Stat.	Probab.
1	TSPREAD does not Granger Cause GDPCH	110	7.29281	0.00805
	GDPCH does not Granger Cause TSPREAD		4.60339	0.03417
2	TSPREAD does not Granger Cause GDPCH	109	6.24964	0.00273
	GDPCH does not Granger Cause TSPREAD		4.35234	0.0153
3	TSPREAD does not Granger Cause GDPCH	108	5.23773	0.00212
	GDPCH does not Granger Cause TSPREAD		1.66559	0.17919
4	TSPREAD does not Granger Cause GDPCH	107	3.05684	0.02025
	GDPCH does not Granger Cause TSPREAD		1.22396	0.30564
5	TSPREAD does not Granger Cause GDPCH	106	1.73825	0.13331
	GDPCH does not Granger Cause TSPREAD		0.68063	0.63919
6	TSPREAD does not Granger Cause GDPCH	105	1.75143	0.11794
	GDPCH does not Granger Cause TSPREAD		0.20243	0.97524
7	TSPREAD does not Granger Cause GDPCH	104	2.22299	0.03955
	GDPCH does not Granger Cause TSPREAD		0.68615	0.68342
8	TSPREAD does not Granger Cause GDPCH	103	1.58693	0.14056
	GDPCH does not Granger Cause TSPREAD		1.30233	0.25324

HOC based causality test results

The HOC based test, proposed in this article, is based on digital whitening. Residuals from the GDP change and Term Structure data are obtained by using higher order moments as explained above. The best ARMA model for a GDP change is found to be ARMA(4,4) . The model parameters (Table 3) are estimated using fourth order cumulants and MATLAB toolbox HOSA Likewise, the best model for the Term spread appeared to be AR(1,4) model, which is presented in the Table 4.

Table 3. GDP ARMA-HOC model

Variable	Coefficient	Std. Error	t-Statistic
C	1.38106	0.146811	9.407083
AR(1)	0.128982	0.068527	1.88222
AR(2)	0.143497	0.080827	1.775364
AR(3)	0.171894	0.055231	3.11225
AR(4)	-0.046561	0.010904	-4.27013
MA(1)	0.274197	0.046255	5.927944
MA(2)	0.242425	0.121984	1.98735
MA(3)	-0.081094	0.018375	-4.41332
MA(4)	0.268694	0.048755	5.51115

Table 4. Term Spread ARMA-HOC model

Variable	Coefficient	Std. Error	t-Statistic
C	1.82815	0.282767	6.46522
AR(1)	0.991982	0.051261	19.35151
AR(4)	-0.151676	0.046951	-3.23052

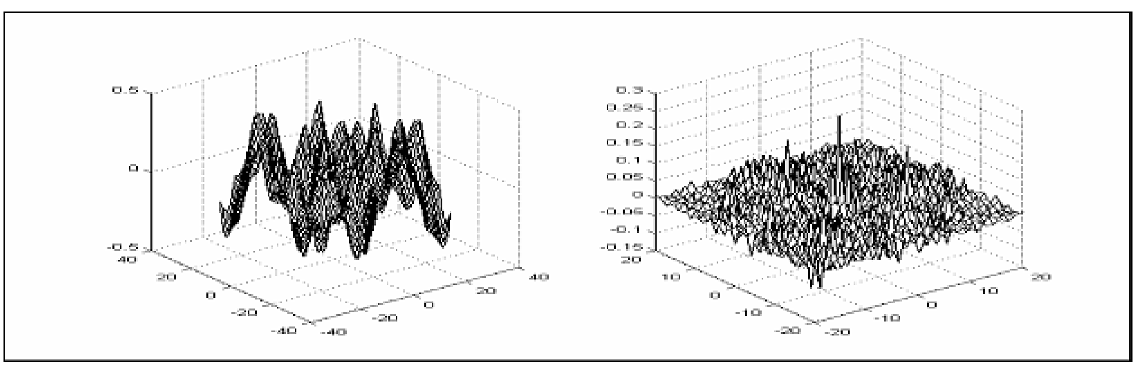
The GDP cumulants and TS cumulants are calculated using equations (3) and (4). Original TS 3-th order cumulants and cumulants of the obtained residuals are presented in Figure 2. Similarly 3-th order cumulants related to GDP variable are presented in Figure 3.

The test states: If there is a statistically significant dynamical relationship between the current GDP residuals and past TS residuals TS causes GDP; If there is a statistically significant dynamical relationship between the current TS residuals and past GDP residuals GDP causes TS. If both hypotheses cannot be rejected, then there is a feedback relationship between the TS and GDP.

Table 3. HOC test results

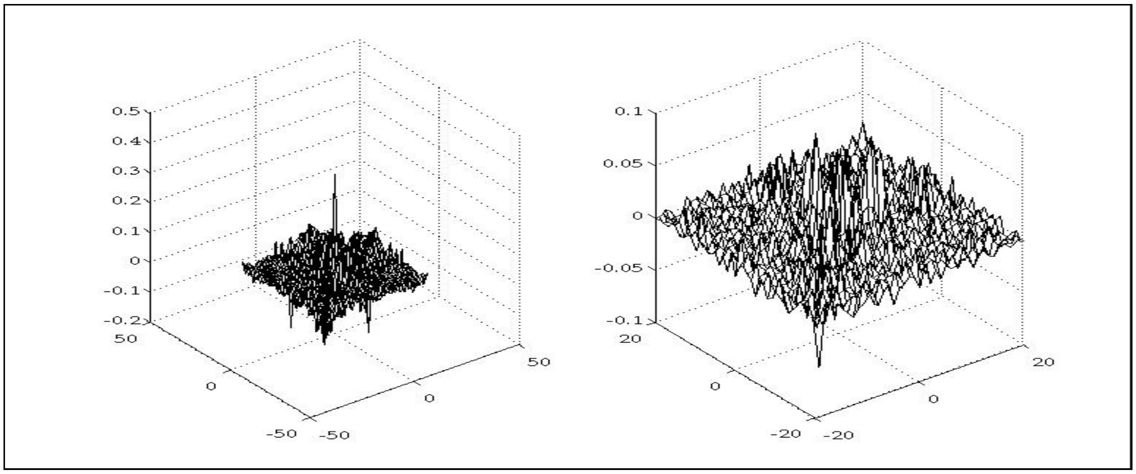
Dependent Variable: RESGDP				
Method:HOS				
Included observations: 108 after adjustments				
Variable	Coefficient	Std. Error	t-Statistic	F
RESTS	-0.061	0.118	-0.517	0.267
RESTS(-1)	0.021	0.113	0.182	0.033
RESTS(-2)	0.298	0.108	2.745	7.533
RESTS(-3)	0.056	0.113	0.499	0.249
RESTS(-4)	0.012	0.004	2.812	4.012
RESTS(-5)	-0.092	0.113	-0.814	0.663
RESTS(-6)	0.224	0.108	2.078	4.319
RESTS(-7)	-0.069	0.109	-0.638	0.913
RESTS(-8)	-0.097	0.108	-0.899	0.663

Figure 2. Third Order TS Cumulants and ARMA-HOC residuals



The results presented in the table 3 strongly prove that innovations or prediction errors of the Term Spread cause the innovations of percent changes of the real US GDP for the lags 2, 4 and 6. For all the other lags, F test shows a non significant causality. Figures 2 and 3 further demonstrate that ARMA-HOC filters captured successfully non-Gaussian properties of the GDP and TS changes . Namely, both residuals have cumulants reduced to zero ,which made application of the new ARMA-HOC test possible

Figure 3. Third Order GDP Cumulants and ARMA-HOC residuals



Conclusion

A new causality test based on HOC (Higher Order Cumulants) is presented in the paper.

The paper further provides two theoretical contributions. Firstly, the proposed test solves the problem of "spurious causality" as a result of the wrong model order selection based on the second order moments, which then necessary leads to colored residuals and the wrong causality lag. The second theoretical contribution is achieved by using higher order cumulants to estimate model parameters and capture non Gaussian properties of the original time series.

To substantiate the analysis, HOC base test was applied to test causality between the Term Spread and real GDP data in the US for the period 1982:q1 -2010:q1. The obtained results clearly show that interest rate spread significantly influences the GDP growth in the second, fourth and sixth quarters.

However, the percentage of the explanation of the GDP growth variability achieved by using the term structure as the explanatory variable in the last two decades is much lower than it was shown in the literature for the period 1970-1990.

There are two possible reasons for this finding: Granger causality test overestimates the coefficient of determination due to the wrong model order or, most probably, the same test doesn't capture higher order moments of the variables that are statistically related. As demonstrated in this paper, the non Gaussian properties of the related variables are captured by the proposed ARMA –HOC test.

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