Measurement of investment portfolio performances

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Abstract

Management of investment portfolio includes technical and fundamental analysis, selection of appropriate securities, designing the optimal portfolio structure, portfolio performance measurement, monitoring and portfolio rebalancing. Investing in securities involves active, balanced or passive investment strategies. In addition, the structure of investment portfolio consists of investment securities and trading book. Investment securities include liquid part that is a secondary liquidity reserve and the income part which is held to maturity in order to achieve the greatest possible profit. Trading book are securities that are traded continuously in order to make a profit according to current price differences. Investment activities are based on information about rates of return, the degree of risk, economic forecasts and risk preferences. After creating an optimal portfolio that is on the efficiency frontier, investment managers perform ongoing monitoring by measuring performance of securities. Investment instruments and tools for evaluating portfolio performance are quite developed: Sharpe's index, Treynor's index, Sortino ratio, Jensen index, Modigliani alpha index, CAPM and APT. Using these indices and models, portfolio managers assess portfolio performance and redesign the portfolio structure to bring current portfolio to the efficiency frontier as close as possible. Usability of these models and techniques are in creating and maintaining an optimal investment portfolio in accordance with the preferences of investors in terms of yield and risk.

Key words: investment portfolio, Sharpe's index, Treynor's index, Sortino ratio, Jensen index, Modigliani alpha index, CAPM and APT

1. Introduction

The investment portfolio is a collection of securities held by investors. The main motive for investing in financial instruments is a profit potential of certain investments: fixed or variable yield and capital gains. In addition, one of the motives of investments in securities is maintaining an appropriate level of liquidity by reducing opportunity cost. So, the portfolio management leads to the collision of two of the three basic investment principles - principles of liquidity and profitability.

Globalization, deregulation and IT development has transformed the economic environment across the globe. Investment in financial instruments reflects the changed economic environment in which financial institutions diversify their activities [1]. In this way, the demarcation line between operations of banks, insurance companies, mutual funds, hedge funds, financial companies is removed. In the competitive "all against all" it is necessary to continuously measuring and monitoring performance of investment portfolios in order to achieve satisfactory profitability at an acceptable level of risk. Financial theory and practice have developed several techniques and models to measure the performance of securities. All of these techniques and models are based on the realized or expected rate of return that will reject certain investments in the period of holding. Also, an important determinant is variance of realized from expected rate of return, measured by standard deviation. Effective portfolio management is based on active, balanced and passive investment strategy in which investors in relation to their risk aversion and trade-off between return and risk trying to create their own portfolio that is optimal, efficient, diversified and compatible with their preferences. They apply a variety of models and technique of portfolio performance evaluation - CAPM, APT, Sortino's index, Sharpe's index, Jensen's index to

create an efficient portfolio - the portfolio at the efficiency frontier with highest level of individual utility. The ultimate goal of portfolio managers is continuous measurement of portfolio performance, monitoring and portfolio rebalancing in order to achieve superior performance in the buying and selling securities.

2. The aim of the paper

The aim of this paper is to outline the basic techniques and models to measure the performance of investment portfolios that are used by banks, mutual funds, hedge funds, insurance companies. These models and techniques are extremely important when making investment decisions. Create and manage an investment portfolio is the multiphase process. First, investment managers create an appropriate structure of the investment portfolio according to their preferences in terms of yield and risk [2]. In relation to risk attitude they implement different investment strategies. The strategy of investment in securities is based on the basic investment principles, respecting trade-off between risk and return, or the maturity composition of the securities. If taken as a criterion trade-off between risk and return, we distinguish between three strategic alternatives: [3]

  a) Active investment strategy - this strategy involves the use of trade account funds in order to achieve profitability. Active approach to portfolio includes cross-purchase of financial instruments, which causes some transaction costs. This strategy appears in two forms:
    o bull strategy - if portfolio manager estimates that price will increase, he will purchase of securities; after price increasing, manager sales the portfolio, achieving profit on the difference between lower purchase prices and more sales price;
    o bear strategy - if we estimate fall in prices, portfolio manager sale portfolio to realize gains between the current higher prices and lower future prices; after price fall, investment manager can retrieve the same position in the securities realizing gains; This strategy produces the active risk.

  b) Structured-active investment strategy (active strategy of risk control) - this strategy is used by investors who invest in stocks, also called the index plus strategy, but investors continue to focus on risk management in relation to the benchmark (e.g. a market index); in this strategy the investor can allocate assets in stocks of companies from different sectors and in line with estimates reallocate funds by selling stocks of companies from one sector and by buying stocks of companies from other sectors; this sector rotation strategy still produces a large active risk, since it involves a highly concentrated investment position in relation to particular sectors; however, it is also a passive-oriented strategy, since the investor's intention to keep the total exposure in relation to investment at the level of benchmark (exposure in relation to the market index).

  c) Passive investment strategy – it involves creating a portfolio with the same risk level as a whole market; the most popular way to balance the portfolio within this strategy is to invest in all stocks that compose a market index; the investment manager does not take into account only the volume but also the structure of investment portfolio: he buys all the financial instruments that constitute a market index, forming the structure of the weights of individual investments in accordance with the weights that individual investments have in the market index; the objective of this strategy is to achieve returns at least equal to the average market yield; this strategy respects the market efficiency hypothesis and allows investors to " coexistence" with the market, or "fate sharing" in terms of profit and risk.

After that, based on the investment strategy, investor takes long and short positions in different financial instruments. In this way, he creates an optimal investment portfolio structure, on the efficiency frontier and at the highest level of utility. The structure of the investment portfolio includes [4]: (a) Investment securities: the liquid part - it's liquid portfolio securities that are often referred to

as secondary reserves; in balance sheet this portfolio is referred as "available for sale". This part of portfolio is composed by government securities, private money market securities and etc. These securities are highly liquid, available for spot sales with minimal losses, to maintain optimal liquidity; (b) Investment securities: the income part – in reports of regulatory agencies it is referred as securities held to maturity. This group includes a variety of securities that investor hold to make a profit. It consists of bonds and notes with a state guarantee, corporate bonds, bonds issued by various government agencies; (c) Trading book account - this part of the portfolio composed by the various securities to increase profitability. These financial instruments are not held to maturity, they are the subject of buying and selling to use the profit opportunity from short-term price fluctuations. The value of the portfolio depends on the portfolio manager current assessment according to prices movement in the future. Here we apply the active investment strategy.

After the portfolio structure built up, the next step is investment portfolio performance measurement, in order to assess the impact of each investment on potential profit. Measuring portfolio performance is extremely important task, given that in a changing economic environment, portfolio performance may vary at any time. These variations can be positive and negative, so it is necessary to determine portfolio performance by measuring whether the existing portfolio close or far from the efficiency frontier. More importantly, based on measurement results investment managers are taking portfolio rebalancing (spot purchase-sale) to bring the portfolio to the efficiency frontier.

3. The research purpose

The main purpose of the research is to improve investment management techniques, in terms of investments in securities. Using different techniques to assess the performance of the investment portfolio, one gets a clearer picture of the securities issuers because some changes in issuer business cause changes in securities prices in the financial markets. In addition, by measuring investment portfolio performance, investors improve basic investment management techniques [5]. On the basis of the measurement results, investors continuously redesign the investment portfolio, promote business efficiency and increase profitability. Increasing the micro efficiency causes an increase in efficiency at the macro level - increasing the financial market efficiency. Namely, based on models and techniques for portfolio performance measurement, the investment analysis becomes more effective instrument in investment decisions making. What is more effective investment analysis, it is more efficient financial markets.

4. Research methods

Investment portfolio valuation is one of the main stages of portfolio management. Managers of banks, investment funds and other institutional investors improve portfolio performance through the various investment strategies. The success of the investment fund is viewed through the degree of diversification, the expected return and risk [6]. Based on such factors, the numerous models have been constructed to assess portfolio performance.

Sharpe's index is an indicator of portfolio performance based on relationship between risk premium and standard deviation of the portfolio. It faces of these two parameters and it is called the rate of prize in yield variability. This ratio implies that the time series of expected returns have a normal schedule. This is one of the main objections to be sent to this portfolio performance measure. Namely, if there is skewness, this portfolio index is not adequate measure of portfolio performance, because then even the standard deviation as a measure of deviation is not effective. Also, in evaluating portfolio performance this index takes into account the total risk, and therefore market and specific risk. Since the process of diversification reduces the non-systemic risk (specific risk), there is a negative trade-off between the degree of diversification and reliability of the Sharpe's index as a measure of portfolio performance: the greater diversification, the lower efficiency of this index.

Is = average portfolio yield - risk-free investment yield / standard deviation

In a graphical presentation of this index we use the capital market efficiency line (efficiency

frontier). It has its "roots" at the point of risk-free investment return, and connects it with the intersection points of return and risk for each portfolio.

Treynor's index measures the portfolio performance in a slightly different way compared to the Sharpe's index. There are two key differences: first, Treynor's index looks at the level of portfolio risk in the context of portfolio yield sensitivity to systemic factors (this elasticity factor of portfolio returns relative to growth in the market as a whole is called the beta factor (β)), second, in a graphical presentation of this index we use a functional relationship between yield and systemic risk (measured by the coefficient β), or instead of the capital market line we use security market line (SML - Security Market Line).

It = average yield - risk-free investment interest rate / β

Sortino ratio is developed by Frank A. Sortino, trying to differentiate "good" and "bad" deviations from the expected return. This differentiation makes it possible to calculate the risk adjusted returns, based only on the negative deviation in yield, while positive deviations are considered absolutely acceptable. Because the positive deviation is preferred, this index "does not punish" portfolio performance based on them, but just focusing on the negative deviation (downward deviation).

Realized portfolio return-Target rate of return (MAR)
S = --
Risk of negative deviations from the target rate

Jensen index - Michael Jensen is 1968 in his paper "The Performance of Mutual Fund in the Period 1945-1964 " announced investment portfolio performance ratio that takes into account the ratio of portfolio performance that are the subject of comparison. Jensen's index establishes "an absolute measure of portfolio performance" as a standard, the investment benchmark for the evaluation of other portfolios. Based on this index is possible to evaluate the performance of portfolio managers. These performances are a function of achieved results compared with target.

αj = Ri - [Rf + x βim (Rm-Rf)]

Ri – the average portfolio yield; βim – the beta of the portfolio; Rf – the risk-free return; Rm - the market index return;

Modigliani risk-adjusted performance (M² or Risk-Adjusted Performance Alpha, RAPA) is a measure of portfolio performance relative to the benchmark. This index compares the risk adjusted portfolio return to a defined reference value. The index measures how much the investor is rewarded for additional risk taking. It shows the ratio between the standard deviation of the underlying portfolio and the comparable portfolio weighted with the average yield difference between the portfolio value and some reference value (eg, risk-free rate of return).

M²*alpha* = x

– the average positive return deviation relative to benchmark;

– the standard deviation of portfolio return;

– the standard deviation of portfolio return deviation relative to benchmark.

In addition to the indices, numerous models of valuation portfolio performances have been developed to improve portfolio management. The two most influential models are CAPM and APT.

CAMP is based on the quantification interdependence between the risk and of equilibrium expected returns, where the overall risk equates with systemic risk. It was developed by Sharpe, Treynor's, Lintner and Mosin's who quantify the relationship between expected and fair rate of return based on risk premiums and risk-free rate of return (eg . yield on government bonds). CAPM model is based on the following assumptions [7]: (1) investors have the same characteristics, except in terms of available resources and risk aversion; (2) capital market is perfectly competitive and individual investors and their transactions can not affect the prices; (3) all investors have the same investment period; (4) investors do not pay tax return and transaction costs; (5) all investors are trying to build a optimal portfolio; (6) all investors are taking the same investment analysis of securities. Taking into account these assumptions, the

CAPM model defines the relationship between the expected portfolio return and portfolio β. Expected portfolio return is equal to the sum risk free rate of return and the difference between the expected yield of the total market portfolio and risk-free rate, weighted with β [7]. Beta is the regression coefficient (slope) securities yield on the market portfolio yield, which represents the sensitivity of securities yield to changes in the overall market.

APT is developed by Ross at the 1976th and it is based also on the quantification of the relationship between return and risk. It discriminates factors on the basis of influence on the yield and risk. This model is based on the law of one price, which suggests that if two assets have the same characteristics, should be sold at the same price [8]. Seen in the context of financial markets, financial instruments that have the same characteristics in terms of return and risk should have the same price. If you make up the imbalance in prices, it would be a current phenomenon that would encourage investors to simultaneous purchase and sale. Arbitration operations would quickly eliminate the scope for profit [3]. In this model, the financial market imbalance is only an exception that confirms the rule: the equal prices of two instruments with the same qualitative characteristics. Arbitration activities are a function of the relationship between the expected returns of two investment alternatives. If there is a discrepancy between the expected returns, the investors sell the asset with lower expected return and buy assets with higher expected return. These arbitration transactions do not require additional money because the revenues from the sale of an asset are equal to the expenditures for the purchase of other assets [9].

In relation to the CAPM that incorporates all factors into a single measure of systemic risk, the operational version of APT models perform separation of factors that systematically affect the portfolio rate of return through factor analysis, and then quantify the impact of these factors. A set of influential factors is difficult to unambiguously define, but quantitative studies have marked some important factors [10]: (1) the industrial production index; (2) the risk premium between securities with different investment grade; (3) interest rates changes; (4) inflation rate changes; (5) unexpected changes of yield curve.

5. Research results

5.1. Sharpe's index

If we create a hypothetical portfolio with four possible scenarios and with equal probability, which would consist of two stocks and one bond, we could calculate the Sharpe's index.

If we assume that the risk-free rate of return is 1%, then the results of this index for each portfolio combination are as follows: S1 = 0,3545, S1;S2 = 0,3640, S2 = 0,3812, S2; B = 1,1111, B = 0,1490, S1; S; B = 1,2327

Based on the obtained results it is possible to evaluate the efficiency of the portfolio. By definition, the higher the index, the portfolio more efficient. This can be proved by observing graphical CML of this hypothetical portfolio.
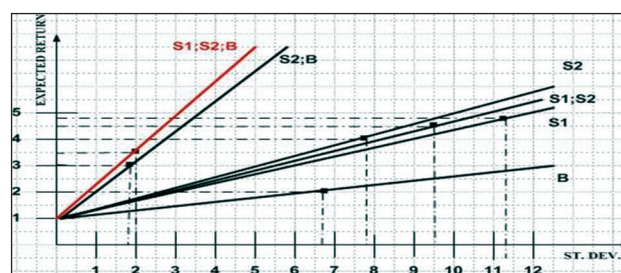

Figure 1. CML of hypothetical portfolios
(Source: Table 1)

Table 1. Hypothetical portfolio of two stocks (S1, S2) and one bond (B)

Scenario	Probability	HPR S1	HPR S2	HPR B	HPR S1; S2	HPR S2; B	HPR S1; S2; B
R	0,25	-12,82	- 8	9	-10,41	0,5	-2,64
P	0,25	17,95	14	-9	15,97	2,5	5,98
N	0,25	8,97	6	5	7,48	5,5	6,49
SQ	0,25	5,77	4	3	4,88	3,5	4,13
Expected return		4,97	4	2	4,48	3	3,49
Variance		125,41	62	45	91,48	3,25	4,083
Standard deviation		11,20	7,87	6,71	9,56	1,80	2,02

Figure 1 confirms the hypothesis about portfolio efficiency. Portfolio whose yield-risk combination has a "steepest slope" is the most effective. This is exactly the portfolio (S1, S2, B) which has the highest Sharpe's index. This portfolio has the biggest yield prize compared to the level of risk.

5.2. Treynor's index

If we assume that the coefficient β for the hypothetical portfolios are 1.00, 0.8, 0.6, 0.4, 0.3 and 0.2, respectively, we can calculate this index and graphically present the SML for each portfolio. With the other data from Table 1, results are as folows: S1 = 3,97%, S1;S2 = 8,70%, S2 = 3,75%, S2;B = 6,66% B = 1,66%, S1; S2; B = 12,45%
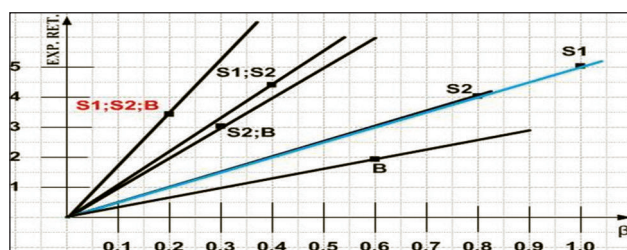

Figure 2. SML of hypothetical portfolios
(Source: Table 1)

This index uses SML lines that start from the origin point towards the northeast, connecting the origin point and the yield-beta combination of different portfolio in the first quadrant of the coordinate system. It measures the angle that every SML lines forming relative to x coordinate and also as the Sharpe's index suggests that the most effective portolio (with best performance) is portfolio that includes all three financial instruments (S1, S2, B). In relation to the Sharpe's index, which is more effective in measuring portfolio performance of stocks, this index is more efficient when portfolio includes stocks and other investment alternatives.

5.3. Sortino ratio

Suppose, that the investment A provides a 5% yield in the first year and 5% yield in the second year. Investment B provides a yield of 0% in the first year and 10% in the second year. The total variance for these two investments is the same, but B investment is more efficient. Therefore, when calculating this index we use as a denominator

variability of return that is below a certain acceptable limit (MAR, minimal acceptance volatility). Denominator in this ratio represents only negative deviations, those that overcome the lower bound (MAR) but MAR is usually 0%. The higher Sortino ratio, the less ability that investment will produce large investment losses. Therefore, more conservative investors prefer investments with the maximum value of this index.

5.4. Jensen index

Jensen index is instructive to estimate abnormal returns relative to a targeted level. The logic of financial markets is as folows: the risky instrument provides a higher yield compared to less risky instruments, or if the financial market is efficient, each investment provides risk adjusted yield. However, if a portfolio provides higher yield compared to the risk adjusted returns, this portfolio has a "positive α", or it provides abnormal returns. The limited value for assessing portfolio performance using this index is 0 (zero): if the portfolio has a index value less than zero, it achieved lower yields than the yield which is expected relative to risk level; if the index value is zero this portfolio provides returns that are expected in accordance with the degree of risk or it has average performance; if the index value is greater than zero, this portfolio provides extra yield.

5.5. Modigliani risk-adjusted performance

This index is very instructive for the interpretation of the investment risk comparing the risk-adjusted yield and benchmark values. It suggests that the returns and risks of the portfolio should start "concession for concession" in relation to the benchmark. If a portfolio has five times more average deviation of risk in relation to the benchmark, then the portfolio should have five times more average deviation of return. In this case investor will earn fair risk premium and the yield is adjusted for additional risk.

5.6. CAPM

This model is based on approximations that do not fully correspond to reality in the capital market, but its usability is indisputable. It could be ap-

plied to real world capital markets but it is necessary to make appropriate modifications of initial settings. One of the most important hypothesis that needs to be modified is the "expected rate of return." Substituting the expected rate with realized rate of return we can apply this model to assess the performance of securities, and its issuers.

$$RXT = \alpha x + \beta x + x\ RMT\ \varepsilon xt$$

If we take as example stocks of Comercial bank Belgrade (KMBN) that is listed on the Belgrade Stock Exchange listing and it is part of BELEX15, and a government bond (A12) as risk-free investment, we can create an index model to assess systemic and non-systemic risk.
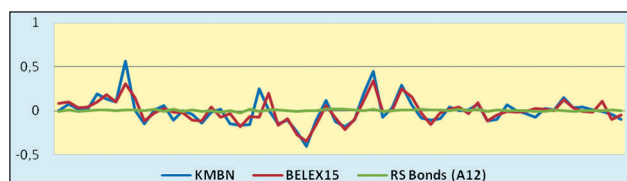

Figure 3. Cumulative monthly returns on KMBN, Belex 15 and government bond (A12)
Source: [11], [12], [13]

Figure 3 shows the cumulative monthly rate of return for the stocks of Commercial Bank, BELEX15 and RS bonds (Series A12) form September 2006 to December 2011th. We see that in the initial period KMBN stocks and market index recorded positive return rates, and after that it became negative. Interestingly, the amplitude of the variation rate of return of these two assets is very high. At the end

of 2009, these assets again achieved positive rates of return, but it is noticeable that these rates are very close to zero yield at the end of period. Government bons provide much lower yield in observed period, but the yield is consistently positive.

Based on informations from the website of the Belgrade Stock Exchange we calculated the rate of return, average return, variance and standard deviation of the portfolio and whole market.

Based on regression analysis of KMBN additional yields in relation to the market index (BELEX15) we obtained the following results.

Statistics of regression is very interesting. All values obtained by regression may be indicated by statistically significant. These results tell us that the correlation coefficient between the extra yield on KMBN shares and market index are 0.62 which indicates a fairly uniform movement of these two parameters. The adjusted R squared tells us that 38.44% variation in additional yields of KMBN can be explained by variations of market index additional yields, while 61.56% of variation can be explained by specific factors. The standard error tells us that this stock has fairly large volatility in observed period. This volatility has a range from ± 0.74%. This stock provided yield below the expected yield in relative to risk level, with very small standard error estimate. Also, with 95% probability it can be confirmed that the alpha is somewhere between -0.057% and -0.054%. Beta coefficient is 1.14 with a very small standard error estimate, while the original beta is somewhere between 1.05 and 1.22. This estimate is based on

Table 2. Descriptive statistics -September 2006 – December 2010 (Source: www.belex.rs)

	Average yield	**Variance**	**Standard deviation**	**Coefficient of variation**
KMBN	-0,000238	0,000989	0,031451	-13191,6
BELEX15	-0,000451	0,000331	0,018193	-4032,31
Gov. Bond	0,055994	0,000108	0,010390	18,55

Table 3. Regression analysis results of KMBN stock (Source: www.belex.rs)

R	0,620475					
R²	0,384989					
Adjusted R²	0,384425					
Standard error	0,745582					
Number of observations	1090					
	Koef.	**Std. E.**	**t-stat.**	**p-value**	**- 95%**	**+95%**
Intercept	-0,055720	0,000792	-70,3804	0,00	-0,572773	-0,054166
Slope	1,136868	0,043523	26,1214	0.00	1,051471	1,222265

a 95% probability. We can conclude that it is an investment (share), whose yields vary according to the cyclical market fluctuations, since it is a beta greater than 1.

5.7. APT

APT is an additional instrument of investment analysis that provides more efficient portfolio management. Although based on factors that are difficult to determine, this model has improved the analytical tools to define expected portfolio return. More broadly, this model is another useful tool for improving the efficiency of financial markets.

6. Discussion

The research results show that it is possible to apply different indices and models to assess the investment portfolio performances. All performance measures are based on yield, expected rate of return, standard deviation, regression and correlation analysis. In terms of indices for measuring portfolio performance, the higher the index value suggests that the portfolio more efficient. Also, based on the index can be made a comparative analysis of portfolio relative to portfolio benchmark. If a portfolio has a greater risk than the benchmark, it should more profitable [14]. Results of regression analysis show that the highest level volatilonosti in yield can be attributed to business factors related to the issuer. This is logical, given that increasing number of securities (diversification of investment portfolio) can significantly reduce non-systemic portfolio risk . Investment managers use these indexes and models to improve the investment portfolio quality. Based on these models and techniques it is possible to evaluate the investment portfolios performances and the investment managers efficiency.

7. Conclusion

The assessing of investment portfolio performances is the condition sine qua none of successful investment management. Since the measuring portfolio performances tools are well developed, investment managers and investment analysts may get clearer results in terms of quality invest-

ment portfolio by using different models and techniques. All models and techniques are based on the expected and the realized rate of return, deviation actual yield from the expected yield, regression and correlation analysis. By applying these techniques its promote portfolio management and financial market is becoming more efficient itself. Based on the index it is possible to make a comparative analysis of portfolio performance against a benchmark portfolio. If two investment portfolios have the same risk level, but the first one has a lower profitability compared to the benchmark, it is necessary to redesign the investment portfolio in accordance with the return-risk preferences. Based on the results of performance measurement it is possible to evaluate the quality of investment managers decision making. Also, using the CAPM model it is possible to estimate the investment portfolio performances, but also to separate the factors affecting the volatility of returns to the systemic (market) and non-systemic (specific) factors. This is very important, since it is based on this model we can monitor the effectiveness of the portfolio diversification.

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